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Optical reflection and mechanical rebound: the shift from analogy to axiomatization in the seventeenth century. Part 1

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Abstract. This paper aims to show that the seventeenth-century conception of mechanics as the science of particles in motion founded on universal laws of motion owes much to the employment of a new conceptual resource – the physics of motion developed within optics. The optical analysis of reflection was dynamically interpreted through the mechanical analogy of rebound. The kinematical and dynamical principles so employed became directly applicable to natural phenomena after the eventual transformation of light's ontological status from that of an Aristotelian 'quality' to a corpuscular phenomenon, engendered by the rise of atomism during the first half of the seventeenth century. The mechanization of light led to a conceptual shift from the analogical employment of dynamical principles in the physical interpretation of reflection to the mechanical generalization of optical principles – the direct application of kinematical and dynamical principles of reflection to mechanical collisions. This first part of the paper traces out the first conceptual shift from Aristotle's original analogy of reflection as rebound to its full concretization. A second part will trace out the second conceptual shift, from the full concretization of this analogy to the axiomatization of already generalized kinematical and dynamical principles of reflection into laws of nature and of motion.

Introduction

> Why is it that objects which are travelling along, when they come into collision with anything, *rebound* in a direction *opposite* to that in which they are naturally travelling, and at *similar angles*. Is it because they move not only with the impetus which accords with their own nature but also with that which is due to the agent which throws them? … Now *every object rebounds at similar angles*, because it is travelling to the point to which it is carried by the impetus which was imparted by the person who threw it; and at that point it must be travelling at an acute angle or at a right angle. … *As then in a mirror* the image appears at the *end of the line* along which the *sight travels*, so the *opposite* occurs in moving objects for they are *repelled* at an angle of the *same magnitude* as the angle at the apex … and in these circumstances it is clear that *moving objects must rebound at similar angles*.[1]

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1 The Pseudo-Aristotelian *Problemata*, Chap. XVI.13; emphasis added. See *The Works of Aristotle, Volume VII: Problemata* (tr. E. S. Forster), Oxford, 1971. For discussions on its Peripatetic provenance between the second and sixth centuries AD see A. Blair, 'The *Problemata* as a natural philosophical genre'; and J. Monfasani, 'The pseudo-Aristotelian *Problemata* and Aristotle's *De Animalibus* in the Renaissance', both in *Natural Particulars: Nature and the Disciplines in Renaissance Europe* (ed. A. Grafton and N. Siraisi), Cambridge, MA, 1999, 171–204.

Here is the ancient Greek claim that rebounding bodies obey the optical law of reflection, with an appeal to the geometry of image location as an explanation for such motion. This constitutes an analogy of *rebound as reflection*. The analogy of *reflection as rebound* had been made much earlier by Aristotle in his analysis of echoes: what is called rebound in relation to sound is called reflection in relation to light; that is, repercussion.[2] Of the two, the analogy of rebound as reflection most clearly sanctions the applicability of geometrical optics to the collision of bodies, while the analogy of reflection as rebound most clearly sanctions an interpretation of the relevant geometry in terms of physical projection. Taken together they sanction the gradual development of a dynamical theory of rectilinear motion and collision within optics. With the anti-Aristotelian mechanization of light in the early seventeenth century, this mature dynamical theory of rectilinear light motion and collision also became generalizable to material rectilinear motion and collision. This transition from the optical tradition to mechanics has three main stages: the introduction and development of the mechanical analogy of reflection within optics, the concretization of this analogy by the generalization of principles of reflection to material bodies in rectilinear motion, and the identification and axiomatization of certain optical principles generalizable to all instances of impact/impulse. This paper is designed to guide the reader along this path in order to reveal the extent to which the new science of mechanics was built upon light.

The optical tradition

Let us begin with the above analogy of rebound as reflection. Though there was no accompanying diagram, the author was undoubtedly familiar[3] with this figure for image location (Figure 1[4]). Here we have the mirror *GD*, the eye *B*, object *A*, the reflected visual ray *BDA* and the image *E* appearing at the end of the line of sight *BDE*. The meaning of the above claim now becomes clear. Since an object in rectilinear motion *BD* able to continue in a straight line through the mirror to *E* (the image's location) would make the angle *GDE behind* the mirror surface, its repelled motion will make the same angle *in front of* the mirror surface as do similarly repelled visual rays, as *GDA*. In other words, since a moving body cannot reach the point *E* due to the obstacle, it rebounds to the opposite point *A* with the same angular relation to the surface.[5] This abstraction from physical differences between a ray and a moving body in

2 See Aristotle, *On the Soul*, II.8 419b2 ff., in *The Complete Works of Aristotle*, *Vol. 1* (ed. J. Barnes), Princeton, 1994; and *idem*, *Posterior Analytics*, II. 15. 98a27, in *The Works of Aristotle, Vol. I* (ed. W. D. Ross), Oxford, 1968.

3 Either from Euclid's *Catoptrics* or Hero of Alexandria's *Catoptrics*. See G. Irby-Masssie and P. Keyser (eds.), *Greek Science of the Hellenistic Era: A Sourcebook*, London, 2002, 195.

4 Taken from Euclid's *Catoptrics*. See K. Takahashi, *The Medieval Latin Traditions of Euclid's* Catoptrica*: A Critical Edition of* De Speculis *with an Introduction, English Translation and Commentary*, Kyushu, 1992, 155.

5 I therefore disagree with Sabra: 'the author of this passage seems to be concerned to bring out a *contrast*, rather than an analogy, between the appearance of images in mirrors and the rebound of objects ... We are told that the object returns at an angle equal to the angle of incidence, but we are not told *why* this must be so'. A. I. Sabra, *Theories of Light from Descartes to Newton*, London, 1967, 69; original emphasis. This very

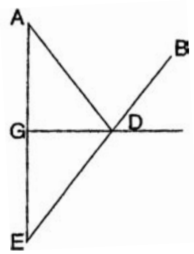

Figure 1. Euclid: reflection and image location.

favour of a purely geometrical identification of their rectilinear motions represents an important conceptual development. We shall now turn to the optical tradition of treating reflection (and refraction) as an impact phenomenon, which greatly reduces the conceptual distance between mechanical rebound and optical reflection created via Aristotelian physics and a non-kinematical notion of light.[6]

Hero and Ptolemy

Hero of Alexandria's *Catoptrics*[7] introduces the core mechanical analogy of reflection into the optical tradition itself via analogies between projectile motion and visual rays. He links the dynamics of physical projection to an object's tendency to rectilinear motion, arguing that what moves most swiftly moves rectilinearly.[8] He provides an atomistic account of matter through which he seeks to make the compactness of walls comparable with that of mirrors. Just as stones or balls do not find entry through the compact surfaces of walls, so neither do rays upon mirrors.[9] Moreover, he ascribes a force of motion to the stone and argues that the force of motion in the case of visual flux also causes its rebounding according to the law of reflection at equal angles, for the combined paths of incidence and reflection are always the minimum possible distance between eye, surface and object. Swift physical projection entails not only visual rays' rectilinear motion but also their reflection at equal angles on encountering a compact obstacle.[10]

Early in his *Optics*,[11] Ptolemy extends this mechanical analogy of visual rays by providing an explanation for the qualitative differences in acuity of vision and intensity

appeal to the geometry of reflection and image location is the reason why the object rebounds in this way. Just as the visual ray rebounds on being prevented from following its straight-line path beyond the mirror, so the object rebounds on being prevented from following its straight-line path beyond the obstacle.

6 Aristotle, *Sense and Sensibilia*, 6. 446a25 and 446b28, in *The Complete Works of Aristotle*, *Vol. 1* (ed. J. Barnes), Princeton, 1994; idem, *On the Soul*, II.7.418b14, in ibid.

7 See M. Cohen and I. E. Drabkin (eds.), *A Source Book in Greek Science*, New York, 1948, 261–5; and Irby-Masssie and Keyser, op. cit. (3), 193–6.

8 Cohen and Drabkin, op. cit. (7), 263.

9 Cohen and Drabkin, op. cit. (7), 263–4.

10 Cohen and Drabkin, op. cit. (7), 263–4.

11 Ptolemy's *Optics*, *c.*160 AD. See A. M. Smith, *Ptolemy's Theory of Visual Perception: An English Translation of the Optics with Introduction and Commentary*, in *Transactions of the American Philosophical*

of illumination in terms of their rays' angles of incidence or impact (as opposed to differences due to a greater or lesser number of such rays). The variable intensity of such rays is explained as instances of a general physical principle governing the variable impact forces of falling bodies.[12] Later, he analyses the resistance which obstacles offer to projectiles in perpendicular and tangential motion in terms of perpendicular resistance (or opposition) to explain the physical basis of the geometrical law of reflection:

> The *relationship of equality* between the angles ... is easily adduced, as is the fact that it *follows a natural course*. For projectiles are scarcely obstructed by objects they strike at tangents, whereas they are obstructed to a considerable extent by objects that resist them [directly] along the line of projection. Accordingly, when anything stands *directly* in the way of these projectiles and stoutly resists them, it interrupts and *opposes the line of projection* extending to the origin [of motion], just as walls obstruct balls that strike them *at right angles*. But obstacles [disposed at a tangent] pose *no* obstruction at all, just as curved bucklers do not obstruct arrows. It should be borne in mind that this explanation *extends to all sorts of moving objects*, and it should be understood that they *all* act in such a way. The *action of the visual ray itself must therefore follow this rule*, and any of the rays that approaches a mirror and then bounces back from it must maintain the disposition that occurs in the paradigm case – i.e. that the angle formed by the line normal to the mirror at the point of reflection and the line of incidence be equal to the angle formed by the same normal and the line of reflection.[13]

Ptolemy here proposes a physical explanation for the equality of angles in terms of the degree of resistance projectiles encounter on impact. Perpendicular incidence entails maximum resistance, and the object rebounds back along the same line of projection (for example, a ball directly against a wall). Tangential incidence entails no resistance, and the object's motion parallel with the obstacle's surface is unaffected (for example, an arrow along a bow-handle). In oblique incidence, the degree of resistance determining a projectile's rebound angle from the obstacle is determined by its incident angle upon the obstacle. Taking this physical explanation as a general rule covering 'all sorts of moving objects', Ptolemy includes the motion of a visual ray, such that the magnitude of its deviation from impact at right angles to the obstructing surface is also the magnitude of its deviation from rebound at right angles to the obstructing surface.[14]

Society (1996), **86**, 2. Smith argues (15) against the influence of Hero's *Catoptrics* upon Ptolemy because (a) whatever he might have found in Hero's *Catoptrics* could also be found in Euclid's, and (b) he failed to mention Hero's 'minimum-line' proof of the law of reflection. However, (a) the mechanical analogies of reflection and rectilinearity are not to be found in Euclid's *Catoptrics*, though they are in Hero's, and (b) Ptolemy sought a physical explanation of the geometrical law of reflection applicable to both the rebound of bodies and the reflection of rays, whereas Hero's geometrical proof of minimum lines offers no such physical understanding of the phenomena of rebound and reflection, or of the law itself, and sharply contrasts with his use of mechanical analogies. Of course, Ptolemy was under no obligation to introduce Hero's metaphysical explanation (a principle of Nature's economy) for what he sought to explain via general physical principles. For Hero on this principle see Irby-Masssie and Keyser, op. cit. (3), 195. It is reasonable to assume, with Smith, Ptolemy's knowledge among the Aristotelian corpus of *De anima* and *Problemata*; see Smith, op. cit., 11.

12 Smith, op. cit. (11), 76–7.

13 Smith, op. cit. (11), 139; emphasis added.

14 Smith mentions that 'the likening of optical reflection to physical rebound is found not only in Hero's Catoptrics, but also in the Pseudo-Aristotelian Problemata', but makes no mention of the conceptual difference between Ptolemy's development of the analogy and the *Problemata*'s simple analogical appeal to the geometry of image formation; see Smith, op. cit. (11), 139.

Here, reflection is taken to be the paradigm case of rebound and must therefore follow exactly the general rule governing such physical phenomena, i.e. the geometrical law of equal angles.

Ibn al-Haytham

The physico-mathematical analysis of reflection and rebound in terms of physical projection was further developed via Ibn al-Haytham's dynamical approach in his *Optics*.[15] In rejecting the extramissionist theory of vision and its concept of visual rays,[16] al-Haytham focuses on the reflection of physical light. His dynamical analysis of reflection as a rebound phenomenon thus becomes fully applicable to material rebound. Like Hero, al-Haytham bases his physical explanation of reflection at equal angles upon the force of rapid motion persisting in light and its encounter with a resisting surface.[17] Like Ptolemy, he appeals to the perpendicular rebound of a ball-like body in the case of natural, downward motion; however, his example of violent (projectile) motion is an arrow being shot horizontally against a mirror; that is, against the very same polished surface from which light rebounds.[18] This clearly indicates that his dynamical explanations of rebound and reflection are conjoined. He then elaborates upon Ptolemy's conception of oblique incidence as a mean between two orthogonal extremes, positing that motion along this oblique line has two components: one perpendicular to the repelling surface, the other parallel to it. He first explicates this via implicit reference to the geometry of image location; that is, as if the body were to pass through the obstacle's surface unhindered (see Figure 1). However, unlike the *Problemata*'s appeal to similar angles formed above and below the surface line, al-Haytham here argues that the oblique motion of a body or light ray below the surface would maintain the same perpendicular and parallel components it has above the surface:

> When the body descends along a *sloping* line, the line of descent falls *between* the *perpendicular* to the surface of the polished body (passing through that body) and the line in the surface *orthogonal* to this perpendicular. And if the motion were to penetrate beyond the point of its incidence, finding free passage, this line would fall between the perpendicular passing through the body and the line in the surface orthogonal to the perpendicular, and it would *maintain the same measure of position* with respect to the *perpendicular* passing [through the body] and the other line *orthogonal to that perpendicular*.[19]

Al-Haytham's clear geometrical formulation of Ptolemy's physical intuitions of the forces involved in oblique rebound is original. How he achieved this is unknown, yet a kinematically identical analysis of compound rectilinear motion within the Greek

15 Though there is doubt about the exact date at which Ptolemy's work was translated into Arabic, it was certainly available to Ibn al-Haytham. See A. I. Sabra, *The Optics of Ibn al-Haytham, Books I–III on Direct Vision*, 2 vols., London, 1989, ii, 58–9, 74.

16 See D. C. Lindberg, *Theories of Vision from Alkindi to Kepler*, Chicago, 1976, 65.

17 F. Risner (ed.), *Opticae Thesaurus …*, Basel, 1572, 112–13, quoted in E. Grant (ed.), *A Source Book in Medieval Science*, Cambridge, MA, 1974, 418.

18 Risner, op. cit. (17), 112–13, quoted in Grant, op. cit. (17), 418.

19 Risner, op. cit. (17), 112–13, quoted in Grant, op. cit. (17), 418; emphasis added.

tradition was available to al-Haytham through Hero's *Mechanica*:[20] this is the parallelogram rule.[21] Hero resolves a geometrical point's motion along a diagonal line into two perpendicular components, and also applies this at a specific location (position) along the line. The compatibility of the parallelogram rule's representation of diagonal motion and the geometry of oblique incidence makes al-Haytham's linking of the two a reasonable explanation for his geometrically refined analysis of oblique incidence and reflection/rebound within optics. In this way, al-Haytham would effectively transfer the parallelogram rule's kinematical analysis of a geometrical point's rectilinear motion from the geometrical foundations of ancient Greek mechanics to the optical analysis of rectilinear impact and rebound/reflection, positing a dynamical interpretation of the rule's perpendicularly related, rectilinear components of motion.[22]

Al-Haytham is thus able to conduct a sophisticated analysis of rebound/reflection. Firstly, he interprets the components of oblique rectilinear motion as dynamical magnitudes. Secondly, he demonstrates that the perpendicular component of motion/force in oblique incidence is alone opposed by the surface, which resists the motive force of the body or light ray and so repels it with equal perpendicular magnitude, while the parallel component, since it in no way opposes the surface, is neither diminished nor altered by the impact. Thirdly, he demonstrates that the ratio between a body's/ray's two components of motion/force remains constant both before and after impact; that is, no matter what position on the *line of incidence* one chooses, the one position sharing the same magnitude of perpendicular motion/force on the *line of reflection* will also share the same magnitude of parallel motion/force.[23]

Though this is a thoroughly mechanical treatment of reflection based upon a sophisticated analysis of the rectilinear impact and rebound of material bodies, al-Haytham is careful not to claim a complete identity between rebound and reflection, for rebounding bodies cannot maintain the rectilinear line of reflection due to their natural tendency to fall downwards, whereas light has no such tendency. Light reflection is the paradigm case showing how bodies would move/rebound if they did not possess a heavy nature.[24] Al-Haytham also lacks an atomistic view of light as the locomotion of

20 While the key section of the Pseudo-Aristotelian *Mechanica* (Problem 1) was transmitted to Arabic culture via al-Khazini's early twelfth-century *Book on the Balance of Wisdom* (*Kitab mizan al-hikma*), this was too late for al-Haytham. On the other hand, Hero's *Mechanica* became accessible to Arab mathematicians in the tenth century via Qusta ibn Luqa's *On Lifting Heavy Objects* (*Fi raf al ashya al-thaqila*). See M. Abbatouy, 'Greek mechanics in Arabic context: Thabit ibn Qurra, al-Isfizari and the Arabic traditions of Aristotelian and Euclidean mechanics', *Science in Context* (2001), **14**, 179–248, 183–7.

21 This rule is also in the Pseudo-Aristotelian *Mechanica*: 'Now whenever a body is moved in two directions in a fixed ratio it necessarily travels in a straight line, which is the diagonal of the figure which the lines arranged in this ratio describe', 848b10, in Aristotle, *Aristotle: Minor Works* (tr. W. S. Hett), London, 1963, 337.

22 It can be argued that if the conceptual source of al-Haytham's analysis of rebound/reflection had been the parallelogram rule of Hero's *Mechanica*, he would have employed it directly. However, such a move would have brought the kinematics of geometrical point motion explicitly into the domain of Aristotelian physics, contradicting its classification of rectilinear motion as simple, not compound.

23 Risner, op. cit. (17), 112–13, in Grant, op. cit. (17), 419.

24 'But in the rebound of a heavy body, *when the motion of repulsion ceases*, the body *descends* because of its nature and tends toward the centre [of the world]. However, light, which has *the same nature of reflecting* [as a heavy body], does not by nature ascend or descend; therefore, in reflection it is moved along its initial line

material particles. Rather, it is the rectilinear transmission of its power or form of heat through a material medium, modelled on the action of fire upon the surrounding air and nearby objects.[25] Nevertheless, his physico-mathematical treatment of reflection to explain the rebound at equal angles of light and bodies from resisting surfaces represents a high watermark in the conceptualization of collision phenomena unsurpassed until the seventeenth century.

Concretization: light and laws of motion

Harriot: a doctrine of material reflexions

In the late sixteenth century the non-Aristotelian and Neoplatonic natural philosophies of Giordano Bruno and Francesco Patrizi reasserted the explanatory importance of light outside optics by each establishing light as one of their basic theoretical principles.[26] This no doubt influenced the so-called Northumberland School[27] by reinforcing the theoretical importance of optical principles for understanding the (re-emergent) atomistic view of Nature. Initially, the rectilinear propagation of particles and force was seen as the most fundamental issue, so principles of light metaphysics were employed.[28] When the issue of the rectilinear impact/rebound of elementary particles

until it meets an obstacle which terminates its motion.' Risner, op. cit. (17), 112–13, in Grant, op. cit. (17), 419; emphasis added.

25 Ibn al-Haytham's *On Light*, in *Ibn Al-Haytham. Proceedings of the celebrations of 1000th Anniversary Held under the Auspices of Hamdard National Foundation* (ed. M. H. Said), Karachi, 1969, 215–16.

26 Bruno's three basic principles are humidity (spherical, indestructible atoms), dryness (an infinite substratum in which atoms exist and move, usually identified as 'ether') and light (which unites the dry and humid and functions as a principle of motion within atoms related to 'soul'). Patrizi's four basic principles are space (a three-dimensional, infinite container of matter lacking resistance), light (a three-dimensional, infinite filler of space lacking resistance), heat (produced by a special kind of light) and 'fluor' (a matter principle of rarefaction and condensation via expansion and contraction, which is also light). In his 1591 book Patrizi explains that 'just as Aristotle discovered the Prime Mover by way of motion, so … I find it by way of "lumen" and "lux" and then … by way of a Platonic Method I descend to the products of light', quoted in E. E. Maechling, 'Light metaphysics in the natural philosophy of Francesco Patrizi da Cherso', unpublished M.Phil. thesis, University of London (Warburg Institute), 1977. For Bruno see H. Gatti, 'Giordano Bruno's soul-powered atoms from ancient sources towards modern science', in *Late Medieval and Early Modern Corpuscular Matter Theories* (ed. C. Luthy, J. Murdoch and W. Newman), Leiden, 2001, 163–80. For Patrizi see also B. Brickman, *An Introduction to Franceso Patrizi's 'Nova de universis philosophia'*, New York, 1941; L. Deitz, 'Space, light, and soul in Francesco Patrizi's *Nova De Universis Philosophia* (1591)', in *Natural Particulars: Nature and the Disciplines in Renaissance Europe* (ed. A. Grafton and N. Siraisi), Cambridge, MA, 1999, 139–69; L. P. Schrenk, 'Proclus on space as light', *Ancient Philosophy* (1989), **9**, 87–94.

27 This was a group of gentlemen scholars such as Nathaniel Torporley, Robert Hues, Walter Warner, Nicholas Hill and Thomas Harriot, centred around their patron the ninth Earl of Northumberland Henry Percy, who also knew John Dee. See J. Jacquot, 'Harriot, Hill, Warner and the New Philosophy', in *Thomas Harriot: Renaissance Scientist* (ed. J. W. Shirley), Oxford, 1974, 107–28; G. R. Batho, 'Thomas Harriot and the Northumberland household', in *Thomas Harriot: An Elizabethan Man of Science* (ed. R. Fox), Aldershot, 2000, 28–47; S. Clucas, 'The atomism of the Cavendish circle: a reappraisal', *The Seventeenth Century* (1994), **9**, 247–73.

28 For instance, in a manuscript written between 1610 and 1620 Walter Warner defines *vis* as 'an efficient power or virtue which may be called light whether sensible or insensible'. He and Nicholas Hill attempted to marry the new atomistic outlook with the medieval light metaphysics of Robert Grosseteste and Roger

became important, principles of light mechanics came more into focus. Interest in such principles grew with the popularity of 'table billiards' among the aristocracy in Europe around the 1570s and 1580s.[29] Moreover, in 1572 al-Haytham's *Optics* was published for the first time, together with Witelo's work.[30] Someone studying the mechanical principles of rectilinear incidence and reflection/rebound contained in these works might easily see their relevance to the rectilinear impact and rebound of billiard balls. The first person to do so was Thomas Harriot.[31]

By the late 1590s Harriot had carefully studied al-Haytham's *Optics* and Witelo's *Perspectiva*, had begun measuring refraction in water and glass,[32] and declared an intention to 'commence war against the Aristotelians'.[33] Reading al-Haytham's book, Harriot would have found that the Arab scholar had taken seriously Ptolemy's suggested grounding of the geometrical law of reflection in the physics of mechanical rebound,[34] analysing the latter to explain the cause of light's reflection in line with the

Bacon: 'Warner needed a source of local motion, generation and alteration in his atomistic system – medieval theories of *vis radiativa* provided him with a solution'. S. Clucas, 'Corpuscular matter theory in the Northumberland circle', in *Late Medieval and Early Modern Corpuscular Matter Theories* (ed. C. Luthy, J. Murdoch and W. Newman), Leiden, 2001, 181–207, 187 and 196. Clucas also makes a strong case for Nicholas Hill adopting this medieval light metaphysical approach to a corpuscular philosophy in his *Philosophia Epicurea …*, Paris, 1601 (reprinted Geneva, 1619), quoting Hill describing light as one of the 'efficient principles of the universe' which 'insinuates itself into the material parts of the world forming everything', in ibid., 198 ff. See also S. Clucas, 'Mediaeval concepts of force in the atomism of the Northumberland circle', in *Science and Cultural Diversity* (ed. J. J. Saldana), Proceedings of the XXIst International Congress of History of Science, CD-ROM (Sociedad Mexicana de Historia de la Ciencia y de la Tecnologia – Universidad Nacional Autonoma de Mexico), Mexico City, 2005, 3060–73.

29 There are reliable records establishing its popularity among the aristocracy in, for example, France between 1574 and 1579 and England around 1587. See C. Everton, *The Story of Billiards and Snooker*, London, 1986, 9–11, and N. Clare, *Billiards and Snooker Bygones*, Princes Risborough, 1996, 5–9.

30 F. Risner, *Optica thesaurus. Alhazani Arabis libri septem … Vitellonis Thuringopolini libri X …* (Basel, 1572), reprinted with an introduction by D. C. Lindberg, New York, 1972. While Alhazen's work was only printed once, the works of medieval optics were regularly reprinted, even into the seventeenth century. See D. C. Lindberg, *A Catalogue of Medieval and Renaissance Optical Manuscripts*, Toronto, 1975.

31 It is worth considering why Harriot chose to tackle this problem. Lohne suggests that Harriot sought to analyse and refute the 'common belief that in all collisions the angles of incidence and reflexion are equal', something which he would have seen contradicted playing billiards. J. A. Lohne, 'Essays on Thomas Harriot: I. Billiard balls and laws of vollision; II. Ballistic parabolas; III. A survey of Harriot's scientific writings', *Archive for History of Exact Sciences* (1979), **20**, 189–312, 193. This belief undoubtedly grew because of the increasing popularity of the *Problemata* as evidenced by its repeated printing in the sixteenth century. See Blair, op. cit. (1), 189; and Monfasani, op. cit. (1), 232 ff. We should note that Harriot had attended Oxford University, where the 1549 Statutes of Edward VI called for the *Problemata* to be taught (Blair, op. cit. (1), 179). Another suggestion about Harriot's choice of topic is that 'he may have connected it with a corpuscular theory of light and his solution of the refraction problem. It may be that it was for this reason that he made a special point of drawing attention to the general inequality of the angles of reflection and refraction'. J. V. Pepper, 'Harriot's manuscript on the theory of impacts', *Annals of Science* (1975), **33**, 131–47, 141. M. Kalmar suggests a connection between Harriot's experience of billiards and the rise of atomism in his 'Thomas Hariot's "De Reflexione Corporum Rotundorum": an early solution to the problem of impact', *Archive for History of Exact Sciences* (1976), **16**, 201–27, 216.

32 *DSB*, Harriot, Thomas; Lohne, op. cit. (31); Batho, op. cit. (27).

33 Quoted in H. Gatti, 'The natural philosophy of Thomas Harriot', in *Thomas Harriot: An Elizabethan Man of Science* (ed. R. Fox), Aldershot, 2000, 177.

34 Smith, op. cit. (11), 139.

former. Besides the perpendicular composition of motions discussed above, Harriot undoubtedly came across several dynamical principles of mechanical rebound: (i) 'the force of reflected motion will be proportional to the force of incident motion and the force of resistance'; (ii) 'the distance covered by reflected motion relative to the obstacle will be proportional to the distance covered by the incident motion relative to the obstacle'; (iii) 'The second [rebound] motion is acquired by the moveable (or mobile) from [this] same resistance',[35] where the incident motion is annulled; (iv) 'the force of resistance is proportional to the force of the first [incident] motion and proportional to the absence of any [additional] effect of the obstacle'; and (v) resistance/repulsion in rebound acts only along the perpendicular.[36] These dynamical insights concern the rebound of movables against hard surfaces, whether by natural (usually perpendicular) or violent (usually oblique) motion – just as light falls upon reflecting surfaces either perpendicularly or obliquely.

Harriot had also read Kepler's *Optics*,[37] which emphasizes the methodological principle of conjoining the motion of light with the motion of bodies.[38] Kepler gives a dynamical explanation of repercussion covering both light reflection and mechanical rebound,[39] employing the principle of composition of motions,[40] and stresses the equality of action and effect in his physical explanation of the geometrical law of reflection.[41] Crucially, he goes on to provide a geometrical demonstration of his argument, employing both the image location diagram of Euclid's demonstration of the law of reflection in his *Catoptrics* (see Figures 1 and 4) and the generalization of its principles from light to bodies – found explicitly in the Pseudo-Aristotelian *Problemata*[42] and, implicitly, in al-Haytham's *Optics*.

Al-Haytham's and Kepler's physico-mathematical analyses of impact and rebound both involve impact against an immovable obstacle, in which the ideal case of

35 Ibn al-Haytham's *Optics*, Kitab al-Manzir, MS Ayasofia 2448, 353, quoted in R. Rashed, 'Optique géometrique et doctrine optique chez Ibn Al Haytham', *Archive for History of Exact Sciences* (1970), **6**, 271–98, 284.

36 Risner, op. cit. (17), 112, quoted in Rashed, op. cit. (35), 286. Sabra has pointed out the problem for al-Haytham in arguing that the force of rebound is related to the force of incidence, when the latter is abolished on impact, which could be resolved by introducing elasticity. See Sabra, op. cit. (5), 76; and A. I. Sabra, *Optics, Astronomy and Logic: Studies in Arabic Science and Philosophy*, Aldershot, 1994, 551.

37 Harriot's research in optics from 1597 to 1601–2 centred on refraction, leading to his discovery of the correct sine law, later propounded by Descartes and Snell. See Lohne, op. cit. (31), 279 and 283; Batho, op. cit. (27), 38. After reading Kepler's *Optics*, Harriot corresponded with him between 1606 and 1609, propounding a novel conception of refraction as the multiple reflection of light rays from atoms throughout the medium. See A. R. Alexander, *Geometrical Landscapes: The Voyages of Discovery and the Transformation of Mathematical Practice*, Stanford, 2002, 115–25. In 1605 Harriot's patron Henry Percy was imprisoned in the Tower of London, where he installed a billiard table (Lohne, op. cit. (31), 193). Harriot's fusion of optics with atomistic matter theory whilst observing billiard ball collisions may have led him to consider the latter via the former or vice versa.

38 'For demonstrations of … [reflection and refraction] … all philosophers and optical writers establish a certain comparison between physical bodies (and their motions) and light.' *Optics*, 1.6: see W. Donahue (tr.), *Johannes Kepler, Optics: Paralipomena to Witelo and Optical Part of Astronomy*, Santa Fe, 2000, 18.

39 Proposition 18; Donahue, op. cit. (38), 26.

40 Proposition 19; Donahue, op. cit. (38), 27.

41 Propositions 18 and 19; Donahue, op. cit. (38), 26 and 27.

42 Kepler refers to the *Problemata* a little later, in his introduction to Chapter 2; Donahue, op. cit. (38), 54.

mechanical rebound in line with the geometrical law of reflection is clearly manifest. In order to fit the different physical situation of mutual mechanical collisions, where the mechanical rebound of bodies need not follow this optical law,[43] the optically based construction of a theory of mechanical impact required conceptual refinement. Harriot undertook this task by seeking to demonstrate that bodies in collision did not always obey the optical law of reflection, as was commonly thought at the time. In this way, the kinematics and dynamics of reflection constitute the 'first principles' on which Harriot constructed his 'doctrine of reflexions' (*De reflexione corporum rotundorum*).[44] He is the first explicitly to employ the geometry of a parallelogram in analysing perpendicularly related components of oblique material collision. In generalizing this composition of motion from light mechanics to colliding balls, he employs a novel rhomboid parallelogram.[45] He also includes the dynamical principle that the force of impact and resistance acts only along the line perpendicular to the obstacle's surface at the point of impact. This is the above-mentioned principle (v) that Harriot came across in al-Haytham. Harriot's rhomboid parallelogram is the key to his theory. He sets out basic dynamical principles which govern the construction of its diagonal, through which the forces of impact and resistance act. Combining the resultant diagonal with a ball's incident motion allows the parallelogram's completion and hence the ball's path of reflection/rebound.

Harriot states that the rebound motion is composed of two *nutus*: the incident *nutus*, which is only deflected on impact, and a new *nutus* imparted by the other body on impact.[46] Here Harriot rejects al-Haytham's annulment of incident motion, expressed in (iii) above, in favour of its deflection, expressed in Kepler's *Optics*, Proposition 18. The line along which this new *nutus* acts is, of course, the same perpendicular line through which the force of impact/resistance acts in optical reflection, in conformity

43 Where the obstacle can impact upon a body at rest or where both bodies are in motion and yet differ in size or weight and relative speeds. Harriot determined that the mechanical rebound of an incident body does not obey the law of reflection only when the obstacle is immovable. See his motive actions 1 and 2, here Figure 5.

44 See individual papers of Lohne, Pepper and Kalmar, opera cit. (31). Some writers lament Harriot's failure to set out his first principles or assumptions, with Kalmar speculating that 'much or even most of the medieval use of the word "reflexion" of Harriot's title refers particularly to optics. There may be nothing more than an analogy here, but it could indicate a useful question for further study'. Kalmar, op. cit. (31), 142. Yet if we treat Harriot's use of such terms seriously, these first principles can be discovered by looking at what Harriot actually learned about the physics of impact from the physics of 'reflexion'.

45 It differs from Descartes's later rectangular version, undoubtedly taken from the Aristotelian *Mechanica* due to its virtually identical kinematical explication. This is because Descartes's mechanical analogy of oblique reflection only involved the oblique incidence of one body (ball) upon an extended surface, not the oblique collision of two moving bodies (balls). The addition of material variability via size (and speed) in such collisions entailed a level of dynamical complexity which the parallelogram of reflection/rebound from an unmoving surface (mirror, ground, wall) could not accommodate.

46 The meaning of this term is a little problematic and is usually left untranslated. Kalmar interprets it as referring to 'the tendency that a body has to move in a given direction with a given speed, whether that tendency is manifested by an actual motion or not', explaining that the 'primary meaning … is a nodding or downward inclining of the head. It can also refer to the downward movement or tendency of a heavy body to move due to its gravity. Harriot generalised the term to include tendencies to move in any direction and exclusive of any cause. This use of *nutus* is, as far as I know, unique to him'. See Kalmar, op. cit. (31), 204. See also Pepper, op. cit. (31), 138; Lohne op. cit. (31), 276 ff.

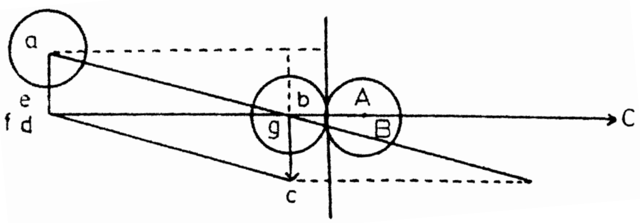

Figure 2. Harriot's motive action one (*c.*1619) (movable obstacle).

with al-Haytham's (v) above. This is the line along which the parallelogram's diagonal is constructed, because the perpendicular component of obliquely incident motion 'determines the forces or active powers of the two colliding bodies'.[47]

The principles employed in the construction of this diagonal are encapsulated in three distinct motive acts, by which Harriot calculates the total quantity of rebound motion. Firstly, Harriot treats the case of equal bodies (or billiard balls) where one is at rest. Here, the ball at rest resists the incident ball with a force equal to the force of incident motion.[48] This principle assumes al-Haytham's (i) and (iv) above, and provides the first part of the diagonal (*bd*, Figure 2). It is this force of resistance that prevents the incident ball from continuing along its line of motion, as if it were travelling unhindered, and is equal to the perpendicular component (*db*) of the incident motion (*ab*).[49] This new line of *nutus* (*bd*) constitutes the diagonal (*bd*), and the parallelogram for this type of collision can be completed (*abcd*): Ball *b* rebounds to *c*, at right angles to the rebound motion of ball *B* to *C*. In line with Kepler's Proposition 19, the perpendicular component of *b*'s motion (*db*), which on impact entails *B*'s rebound motion *BD*, is equal and opposite to *B*'s resistant force *bd*. It is this *nutus* (*bd*) of *B* that, on impact, resists *b*'s motion along its incident line beyond *B* with a perpendicular component equal to *db* (indicated, though unlettered, in Figure 2). Harriot's dynamical understanding of this type of collision corresponds to that of contemporary mechanics (*abc*, Figure 2 = VAV_1, Figure 3; *BC*, Figure 2 = BV_2, Figure 3).[50]

Secondly, Harriot treats the case where the obstacle ball is also moving in line with the other's perpendicular component, and provides equal resistance according to both its equal size (*bd*) and its equal quantity of motion along the perpendicular line (*AB*) (Figure 5).[51] This principle extends al-Haytham's (iv) above, by making allowance for the increase in an obstacle's effect, via its motion. This provides the second part (*de*, equal to *AB*) of the diagonal. Combining these two motive actions gives the diagonal

47 Lohne, op. cit. (31), 202.

48 Lohne, op. cit. (31), 202.

49 Hence the appeal to the geometry of image location and its mechanical application.

50 See C. B. Daish, *The Physics of Ball Games*, London, 1972, 193. Lohne makes the interesting point that Harriot was most likely the first person to state that colliding bodies sometimes move off at right angles to each other (a common feature in billiards) and provide a dynamical analysis of this situation. Lohne, op. cit. (31), 193.

51 Lohne, op. cit. (31), 202–3.

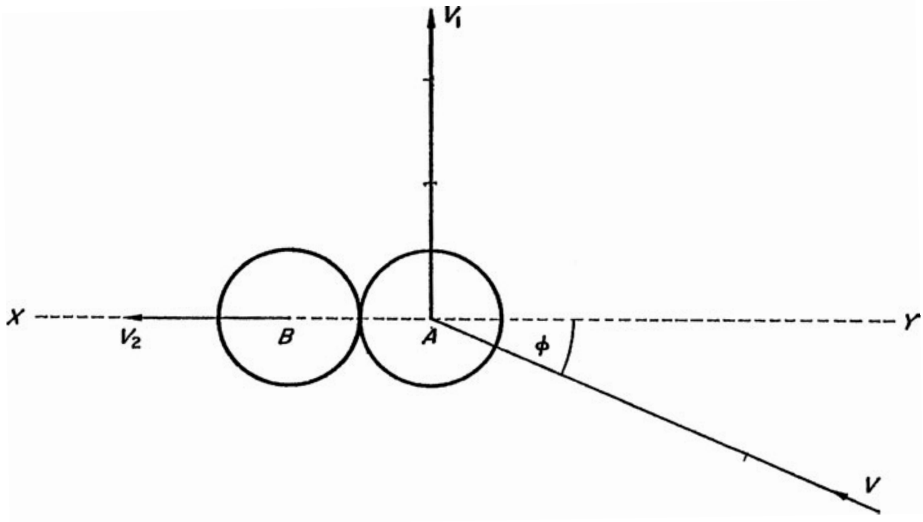

Figure 3. Oblique impact of billiard balls (1972). Simplified diagram from C. B. Daish, *The Physics of Ball Games*, London, 1972, 159.

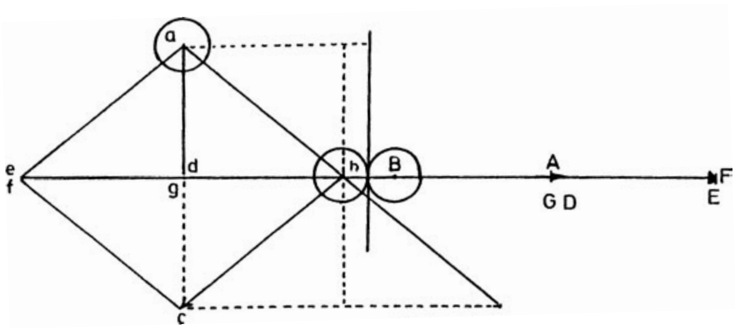

Figure 4. Harriot's motive actions 1 and 2 (moving obstacle).

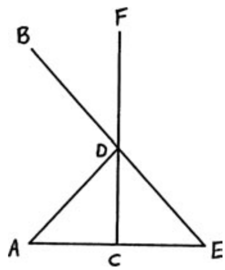

Figure 5. Kepler on mechanical rebound.

be, and the parallelogram for this type of collision can be constructed (*abce*). Ball *b* rebounds to *c* while ball *B* rebounds to *E*. Harriot's reliance on optical principles is evident in his appeal to the geometry of image location and its mechanical application found in Kepler's *Optics* (Figure 4),[52] including the drawing of a tangent line at the point of impact to represent a reflecting surface.[53]

52 As implicitly employed by Pseudo-Aristotle and al-Haytham, and explicitly employed by Kepler. Indeed, the same mechanical argument and geometrical demonstration, taken from the *Problemata*, was further popularized via Josephus Blancanus's *Aristotelis loca mathematica ex universis ipsius operibus collecta et explicata*, Bologna, 1615, 243–4.

53 This is one of three cases of rebound, in which the law of reflection is obeyed, the other two being where the obstacle body is infinitely heavy, and its equivalent case, where the obstacle body is taken to be fixed (see Figure 6). Kalmar, op. cit. (31), 205; Lohne, op. cit. (31), 225.

Thirdly, in treating the case of *unequal* bodies in mutually oblique motion, Harriot develops further al-Haytham's (iv) above, by incorporating into his calculation of forces (or active powers) the effect of size differentiation. The greater body impacts upon the lesser with a force greater than if they had been of equal size – as calculated in the first motive act; so, if it is bigger than the other by, say, one-third, then one-third of the *nutus be* must be added onto it to give *ef*, resulting in the final nutus *bf* (Figure 7).[54] This is because the excess in size would increase both its force of resistance simply as an obstacle at rest (*bd*, due to motive act one) and its force of resistance as a moving obstacle (*de*, due to motive act two). Once this diagonal is completed (*bf*), the parallelogram for this type of collision can be constructed for ball *b* (*abcf*). Ball *b* rebounds to *c*.[55]

Having provided the dynamical and geometrical principles of his 'doctrine of reflexions', Harriot demonstrates three corollaries:

> First in what cases the angles of reflection are equal to the angles of incidence and in what cases they are not. Indeed, they are believed to be equal always, but this is false. Secondly, that each of the bodies, equal or not, has at the finish the same distance from the line of *nutus* as it had at the start. Finally, that the mutual distance of the two bodies is the same at the finish as at the start.[56]

In the first corollary he successfully corrects the then common notion that the geometrical law of reflection holds for all cases of rebound. However, the latter two are derived directly from the physical interpretation of this same geometrical law within the tradition of light mechanics. In the second corollary, this 'distance from the line of *nutus*' constitutes the *parallel* component of obliquely incident motion that is unaffected by *perpendicularly* acting forces of impact and rebound, and is therefore always conserved. This basic principle of light mechanics is clearly expressed in the *Optics* of al-Haytham and Kepler. The third corollary extends al-Haytham's (ii) above to the case of two incident bodies. This principle is itself a mechanical generalization of the law of reflection: the incident ray and reflected ray, seen as the oblique trajectory of a body, show by their equal perpendicular components that the body's first and final positions are equally distant from the reflecting surface, or obstacle (Figure 2). Harriot generalizes this further, first in the cases of mechanical reflection in which the obstacle is effectively fixed (*aA* and *cC*, Figure 6), then in the cases of mutual collision where each moving body is the other's obstacle (*aA* and *cC*, Figure 7).

Finally, let us examine Harriot's analysis of the reflected line *bc* (or *BC*).[57] For Harriot, the reflected motion compounds the deflected ball's incident motion, or *nutus*

54 Lohne, op. cit. (31), 203.

55 Parity of reasoning and the lesser's diminution in size and active power enables the construction of a shorter diagonal (*BF*), its parallelogram (*ABCF*), and *B*'s path of rebound *BC*.

56 Lohne, op. cit. (31), 204.

57 This diverges from the optical case because it covers both equal and unequal angles of incidence and reflection, and employs rhomboid parallelograms whose adjacent sides do not correspond to the perpendicularly related components of oblique motion. Descartes's later use of the rectangular parallelogram for compound motion in oblique impact and rebound considers only the oblique impact of light or a ball against a plane surface, which therefore rebounds in line with the law of reflection.

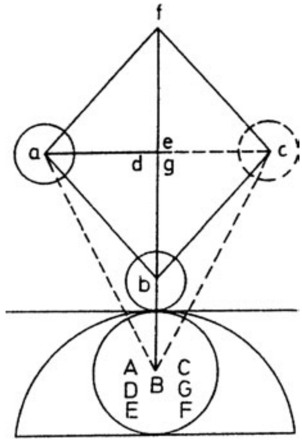

Figure 6. Harriot's motive actions 1 and 2 (immovable obstacle).

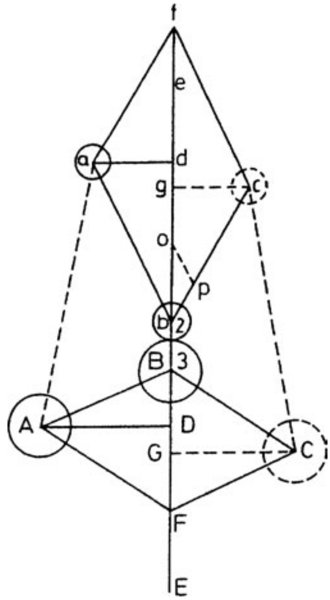

Figure 7. Harriot's motive actions 1, 2 and 3.

(*ab*) and the new *nutus* resulting from impact (diagonal *bf*).[58] He again appeals to the geometry, and mechanical interpretation, of image location (Figure 1) in his interpretation of side *fc* of the parallelogram as the conserved first *nutus* (*ab*):

> The motion due to the first *nutus* no longer takes place in the same line as before but in a parallel line which also in the second time *x* must be of the same magnitude, and signified by the line *fc* which is equal and parallel to *ab*. If there had not been any *nutus* due to the stroke, *fc* would have been a continuation of *ab*, and so much would the body have moved in the second time *x*.[59]

Harriot's method of compounding the two lines of *nutus* is to imagine the ball continuing along line *ab* as if unhindered (as Kepler's line *DE*) while at the same time the force of impact carries the starting point of this second line at *b* along the diagonal to *f*: in any time *t* in which this starting point travels along *bf*, the ball travels in a direction parallel to *ab* to a point on line *bc* (see line *op* in Figure 7).[60] He is careful to clarify that 'the apparent and true second motion [rebound] does not take place in any of the lines *bf* or *fc*, but they denote the ratio of the forces in these two *nutus* which compose the apparent motion in a third line [*bc*]'.[61] This way of compounding the

58 Lohne, op. cit. (31), 202.

59 Lohne, op. cit. (31), 203. Compare with Figure 4 above, of Kepler's Proposition 19. Harriot's line *fc* here corresponds to Kepler's line *DE*.

60 'Hence it follows that at the same time as the body *b* continues its motion from time *f* to *c* and the whole line *fc* is translated as described above, the body's centre will describe a certain third line which when completed will be the line *bc*, drawn from the point *b* to *c*'. Lohne, op. cit. (31), 203.

61 Lohne, op. cit. (31), 203.

two *nutus* ensures that the optical principle of the unchanged parallel component of oblique motion (*ad*, *gc*) is built into the process of parallelogram construction, where such lines represent a real component of oblique motion – since a ball will physically move along this when its perpendicular component is lost on impact (see Figure 2 above).

Conclusion

While Harriot's theory elaborates further on collisions between bodies of varying sizes, this paper has sought to show that his 'doctrine of reflexions' rests upon basic kinematical and dynamical principles of optical reflection. These were developed within the tradition of light mechanics, and subsequently generalized to accommodate rectilinear material impact and rebound from a surface. With the re-emergence of atomism and the appearance of rectilinear collisions between bodies via the billiard table's horizontal plane, Harriot made a creative connection between optics and mechanics. He generalized these optical principles further to accommodate two-body collisions, constructing a geometrical method for their dynamical analysis capable of accommodating their reduction to cases of simple rebound in line with the law of reflection. The slight inaccuracy of his formulae for the rebound velocities of unequal balls, resulting from an error in calculating their respective third actions, is irrelevant in this connection.[62] What is important is Harriot's grounding of his 'doctrine of reflexions' in these principles, with which he analyses the dynamics involved in material collisions.

Given Harriot's more sophisticated use of optical principles to analyse impact phenomena, the issue of his influence upon later mathematicians and natural philosophers should be considered. Though he never published his 'doctrine of reflexions', there were at least two distinct paths of possible transmission. Firstly, Harriot personally gave copies of his 'doctrine of reflexions' (*De reflexione* …) to the Earl of Northumberland and Walter Warner.[63] Warner later became an associate of the

62 Kalmar, op. cit. (31), 214; Lohne, op. cit. (31), 196.

63 Harriot's letter to the Duke of Northumberland, dated 13 June 1619, which accompanied a copy of his *De reflexione* and wherein he talks of perfecting his 'auntient notes of the doctrine of reflections of bodies' and imparting them to Warner. See J. O. Halliwell, *A Collection of Letters Illustrative of the Progress of Science in England*, London, 1841, 45. There is renewed interest in the likelihood of earlier collaboration between Harriot and Warner on collisions (around 1603), due to the discovery of previously unknown manuscripts of Warner's on collisions and motion; see S. Clucas, 'Thomas Harriot and Walter Warner on collisions: English mechanics in the early seventeenth century', paper read at the XXII International Congress of the History of Science, Beijing, China, 24–30 July 2005. Yet the recognition of the applicability of optical principles to material motion, which grounds Harriot's *De reflexione*, is implicit in Henry Percy's listing of 'The doctrine of motion of the optics', written in 1595. See G. B. Harrison (ed.) *Advice to His Son, by Henry Percy Ninth Earl of Northumberland*, London, 1930, 67. Given that Harriot had thoroughly studied al-Haytham's *Optics* in the early 1590s, his 'auntient notes' may well date as far back as then. However, this phrase of Percy's has been ignored by many, including Clucas. See Clucas's 2005 conference paper, cited above, and his 'Thomas Harriot and the field of knowledge in the English Renaissance', in *Thomas Harriot: An Elizabethan Man of Science* (ed. R. Fox), Aldershot, 2000, 93–136, 107. In the latter, Clucas employs the misquotation of 'the doctrine of motions, optics' found in Christopher Hill's *The Intellectual Foundations of the English Revolution*, Oxford, 1965, 142.

so-called Cavendish Circle,[64] and Charles Cavendish himself possessed a copy *of De reflexione*.[65] These facts establish a crucial link between the Northumberland School and the later Cavendish Circle. This Circle opened up an informal English channel for the communication of ideas (especially about light/optics) with European thinkers. Its members (especially Hobbes) and their associates either met or corresponded with figures such as Descartes, Galileo, Gassendi, Mersenne, Mydorge and Roberval in the 1630s and 1640s.[66] Secondly, by around 1670 the librarian of the Royal society, John Collins, had read transcriptions of Harriot's work on collisions,[67] and the society's first president, Lord Brouncker, owned 'two sheets of HARRIOT de Motu et Collisione Corporum'.[68] It was Collins whom the Royal Society asked in 1688 'to make a study of all the authors who had written on "the nature, principles and laws of motion", especially Descartes, Borelli and Marcus Marci'.[69] Yet there was no mention of Harriot's 'reflections of round bodies' in any of the discussions or literature on laws of motion/impact sponsored by the Royal Society. We can therefore plausibly conclude that his influence upon these mathematicians working on laws of motion for direct collisions did not involve direct study of his *De reflexione*. Their basic appeal to statical concepts represents a developmental strand of mechanical thought distinct from the more optical approach stemming from the atomistic concretization of the mechanical analogy of optical reflection, exemplified by Harriot's early attempt to correct the late sixteenth-century belief that mechanical collisions always occur in accordance with the law of reflection. Perhaps all that can be reasonably claimed is that knowledge of his general approach to the study of collisions in terms of *reflections* most probably passed by word of mouth between members and associates of the Cavendish Circle, then to a select group of European mathematicians and would-be mechanicists centred in Paris. Its traceable influence was lost as it flowed into and merged with a rising tide of optico-mechanical thought.[70] In the absence of any direct causal influence, we can say that

64 Mainly William and Charles Cavendish, Thomas Hobbes, John Pell and Robert Payne. Kenelm Digby became associated with the Circle via correspondence with Hobbes. See J. Jacquot, 'Sir Charles Cavendish and his learned friends', *Annals of Science* (1952), **8**, 13–27 and 175–91, and Clucas, op. cit. (27).

65 Lohne, op. cit. (31), 215.

66 Jacquot, op. cit. (64); Clucas, op. cit. (27), 252–9. This explains, for instance, why Kenelm Digby's *Two Treatises* of 1644, containing his arguments for the material nature of light (as a body) and the notion of reflection as violent motion, was first published in Paris, where there was a receptive intellectual environment and where he lived from 1641 to 1645. See Clucas op. cit. (27), 256. Implicit in this mechanization of light is not simply the idea that light should therefore obey the same laws of motion as bodies, but the more fundamental notion that the laws governing material bodies must be based upon laws of light because light motion represents the ideal case of material motion/impact, free from the effects of gravity and air friction.

67 Collins had been described by Isaac Barrow as the 'English Mersenne' (see his *DSB* entry), corresponding with scholars such as Barrow, Newton, Wallis, Borelli, Huygens and Leibniz.

68 Lohne, op. cit. (31), 190 and 215.

69 A. R. Hall, 'Mechanics and the Royal Society, 1668–70', *BJHS* (1966), **3**, 24–38, 28.

70 We must include the conceptual spurs provided in the late sixteenth century and the earlier seventeenth by Aristotle's presentation of the mechanical analogy of reflection in his *Problemata*, the exposition of the parallelogram rule in the Pseudo-Aristotelian *Mechanica*, Kepler's explicit linking of light and material motion in his *Optics* and, of course, Descartes's introductory illumination of his mechanical philosophy via his *Dioptrics*, in which he explicitly applies the said parallelogram rule to reflection and, via the above analogy, to rebound.

Harriot's 'doctrine of reflexions' was ahead of its time and effectively illustrates the potential of this more optical approach to the formulation of laws of motion/impact in the hands of a great mind, a potential fully realized by Newton but glimpsed in others such as Pardies.[71]

The historical significance of Harriot's 'doctrine of reflexions' lies in the fact that it shows the extent to which someone could successfully employ the kinematics and dynamics of optical reflection in constructing a mechanistic theory of (direct and oblique) impact motion at the beginning of the seventeenth century. It constitutes the purest conceptual distillation of the 'reflection as rebound' Zeitgeist achieved before Newton.[72] This was a potent concoction that explicitly analysed the idealized, oblique (rectilinear) motion of bodies in terms of two perpendicularly related independent components, decades before Galileo's similar analysis of (oblique) projectile

71 In 1670 Pardies sought to correct Descartes's erroneous laws of impact by explicitly introducing the dynamics and kinematics of optical reflection, shifting from the analysis of direct to oblique collisions between bodies. See Ignace-Gaston Pardies, *Discours du movement local*, Paris, 1670, translated into English by Henry Oldenburg as *A Discourse of Local Motion*, London, 1670, in *Descartes in Seventeenth-Century England, Vol. 7: Critiques of Descartes* (ed. R. Ariew and D. Garber), Bristol, 2002, 38 ff. There are striking similarities between his, Harriot's and Newton's work of 1666 on mutual rectilinear collisions in their use of the physics of reflection, which deserve a fuller examination. But whilst, unlike Harriot, he includes circular motion in his analysis, Pardies signally fails to extend the parallelogram rule to circular motion, unlike Newton a few years earlier. This is because he does not make Newton's creative conceptual leap of analysing such motion via the geometry, and dynamics, of reflection. It is unlikely that Pardies had access to Harriot's writings, yet he was familiar with the optico-mechanical writings of Maignon and Hobbes.

72 This common view no doubt arose from the widespread dissemination of Aristotle's *Problemata* in the sixteenth century (see Blair and Monfasani, opera cit. (1)) and its specific application of the geometry of image location to mechanical rebound, which was popularized in 1615 by Josephus Blancanus (op. cit. (52)). Moreover, the Latin publication of Ibn al-Haytham's light-mechanical treatment of reflection and refraction in 1572 would have reinforced interest in exploring the dynamics of reflection and mechanical collision, given the imminent rise of an atomistic conception of light. In 1624 Jean Leurechon noted that 'hence by mathematical principles, the games of Tennis may be assisted, for all the moving in it is by right lines and reflections', and that 'the maximes of reflections cannot be exactly observed by locall motion, as in the beames of light and other qualities, whereof it is necessary to supply it by industry or by strength otherwise one may be frustrated in that respect'. Leurechon, *Récréations mathématiques* …, Paris, 1624, Problem 71 (there were twelve editions before 1630 and more after); see also the 1654 English edition, W. Oughtred, *Mathematical Recreations* …, London, 1654, 122–4, and also 1633 and 1674. Interestingly, Descartes refers to it in a letter to Mersenne, in April/May 1634, and this Problem 71 seems to inform Discourse 2 of his later *Dioptrics*, while Mydorge's first major writing in 1630 (reprinted 1643) was a review of *Récréations mathématiques* (see *DSB*, Mydorge). Seventeenth-century appeals to the law of reflection in analysing (especially oblique) mechanical impact can also be found in Marcus Marci, *De proportione motus* …, Prague, 1639, N-3; Kenelm Digby, *Two Treatises: One Concerning Bodies* …, Paris, 1644, 45; E. Torricelli, *De motu gravium* …, Florence, 1644 (see *Opere di Evangelista Torricelli* (ed. G. Loria and G. Vassura), 4 vols., Faenza, 1919, ii, 230); G. B. Baliani, *De motu naturali gravium* …, Genoa, 1646, 105; E. Maignan, *Perspectiva Horaria* …, Paris, 1648, 292 ff. (including Lemma III on the equality of a rebounding ball's impulse and repulse, 297); W. Charleton, *Physiologia Epicuro-Gassendo-Charletona* …, London, 1654, 471–4 (compare P. Gassendi, *Syntagma philosophicum*, in *idem*, *Opera omnia*, 6 vols., Lyons, 1658, i, 354, 360–1); T. Hobbes, *De Corpore* …, London, 1655 (see the English translation of 1656 in *The English Works of Thomas Hobbes of Malmesbury* (ed. W. Molesworth), 11 vols., London, 1839–45, i, 384); and *idem*, *Problemata physica*, London, 1662, in which he provides an update of al-Haytham's claim that a heavy body and a light one share the same nature of reflecting until gravity alters the body's motion via discussion of a bullet shot against a wall (see English translation of 1682, *Seven Philosophical Problems*, in *English Works* (ed. Molesworth), vii, 50 ff.); and Pardies, op. cit. (71), 38 and 46 ff.

motion[73] and the dynamically simpler theories of direct impact propounded fifty years later by Wallis, Wren and Huygens.[74]

This paper has shown how Aristotle's original development of the mechanical analogy of reflection eventually led to its atomistic concretization in Harriott's 'doctrine of relexions', through the development of a projectile model of light motion within optics which allowed the kinematics and dynamics of impact/collision to be constructed outside the prohibitive framework of Aristotelian physics. A second part will show how Harriot's light-mechanical 'high-watermark' was entirely transcended by the selective axiomatization of such generalized kinematical and dynamical principles developed in the physical interpretation of optical reflection. This part having traced the first conceptual shift, the light-mechanical generalization of optical principles to material collisions, the argument of the second part will trace out the next conceptual shift, from Harriot's optically grounded theory of impact to the creation of optically grounded laws of nature/motion by Descartes and Newton.

73 This analysis could be broadened by considering the convergence of geometrical treatments of oblique motion in both traditional mechanics and optics. The analysis of bodies moving on an inclined plane serves to idealize physical motion of a falling body in a similar way to the horizontal plane of a billiard table idealizes the motion of colliding bodies, where the geometrical representation of perpendicularly related components of such oblique fall is identical to that of oblique optical incidence. See Galileo Galilei, *Dialogues Concerning Two Chief World Systems* (tr. S. Drake), Berkeley, 1967, 23; and *idem*, *Two New Sciences* (tr. H. Crew and A. de Salvio), New York, 1954, 181 ff. As a foundational mathematical discipline optics was studied by all late sixteenth-century mathematicians. After Risner's Latin edition of Ibn al-Haytham's *Optics* in 1572, they would have been very familiar with its physical-projection interpretation of the geometry of optical reflection and dynamical analysis in terms of perpendicularly related and independent components of motion/force. In 1588, well before his work in mechanics and kinematics, Galileo had applied to the Academia del Disegno in Florence for the post of professional geometer to teach artists linear perspective, among other disciplines, probably using Daniel Barbaro's *Practica della Perspectitiva*, published in Venice several times in the 1560s and consulted by members of the Academia. See S. Edgerton, *The Heritage of Giotto's Geometry: Art and Science on the Eve of the Scientific Revolution*, Ithaca, 1991, 224 ff. In 1594 Galileo had also visited Guidobaldo del Monte to view his as yet unpublished *Perspectiva libri sex*, Pesaro, 1600. By 1601 Galileo was teaching optics; see S. Drake, *Galileo at Work*, Chicago, 1978, 35 and 51; and S. Dupre, 'Mathematical instruments and the theory of the concave spherical mirror: Galileo's optics beyond art and science', *Nuncius* (2000), **15**, 551–88. The relevance of optical principles for the conceptualization of circular and projectile motion as resulting from two independent component motions was explicitly raised in the seventeenth century by Stefano degli Angeli. Arguing against Riccioli's erroneous analysis of projectile motion he said, 'As for the mixture of two or more motions, nothing is easier. They always mix or, if we prefer, nature is perfectly able to hold them apart, so that the effects of the one do not impair or hinder those of the other, as Kepler and Descartes have shown it occurring in the case of the reflection of light' (refering to the *Optics*, and *Dioptrics*, respectively, both published before Galileo's *Two New Sciences*). S. degli Angeli, *Considerationi sopra la forza di alcune ragioni fisico mattematiche …*, Venetia, 1667, Quartae Considerationi, 20, quoted in A. Koyré, *A Documentary History of the Problem of Fall from Kepler to Newton*, Philadelphia, 1955, 395.

74 The common feature of these three analyses of mutual collision, conducted to determine 'laws of motion', is that they are all concerned with the dynamically simpler case of direct impact. This allows Huygens, Wren and Wallis to conceptualize such phenomena via appeal to statical principles, the balance or lever. See Hall, op. cit. (69), 24–38; and R. Dugas, *A History of Mechanics*, New York, 1988, 172 ff.